Abaye said: You will also find that the open space is one quarter of the total area in a city that is a thousand cubits by a thousand cubits. How many cubits is the extended boundary of the city without the corners? It is eight million square cubits. Additional areas are appended along each side of the city and extending two thousand cubits beyond the city itself. Each of these areas is two thousand cubits by one thousand cubits, for a total area of two million square cubits. Since there are four of these zones, their total area is eight million square cubits. How many cubits are the corners? They are sixteen million square cubits, as squares of two thousand cubits by two thousand cubits are added to each of the four corners.

The Gemara asks: According to this calculation, the eight million square cubits of open space, which is a thousand cubits by a thousand cubits on each side, and an additional four million square cubits from the corners, a thousand cubits by a thousand cubits in each corner, which are connected to the open space. How much is the sum total? It is eight million square cubits.

Subtract four million square cubits of the extended boundary for the area of the open space, which is a thousand cubits by a thousand cubits on each side, and an additional four million square cubits from the corners, a thousand cubits by a thousand cubits in each corner, which are connected to the open space. How much larger is the area of a square than the area of a circle? It is one quarter of the area of the circle. Subtract one quarter from the eight million square cubits of open space, and six million square cubits are left; and six is precisely one quarter of twenty-four.

Ravina said: What is the meaning of the statement that the open space is one quarter? It is one quarter of the boundary. This halakha was indeed stated with regard to a square city. However, there is open space only along the sides of the city but not at its corners. Accordingly, a city that is two thousand cubits by two thousand cubits has a total extended boundary of thirty-two million square cubits, of which eight million square cubits, two thousand cubits by one thousand cubits on each side, is open space. The open space is thus one quarter of the total.

Ravina’s opinion - Ravbeinu Hananel explains that the purpose of Ravina’s statement is to explain that Abaye does not include the area of the city itself, but only the surrounding areas. Rabbi Zeraḥya Halevi explains it completely differently: Ravina maintains that there are only five hundred cubits of open space on each side, which combine to form an area of a thousand by a thousand cubits, and beyond that there is an additional space of a thousand cubits on each side for the extended boundary, which combine to form two thousand by two thousand cubits. The extended boundary does not include corners. Consequently, the open space is one quarter of the area of the extended boundary.
Rav Ashi said the opposite: What is the meaning of the statement that the open space is one quarter of the total extended boundary? **One quarter of the corners.** Open space is granted only in the corners, and not along the sides. Accordingly, the open space is one thousand cubits by a thousand cubits in each corner, for a total of four million square cubits. The total extended boundary in each corner is two thousand cubits by two thousand cubits, or four million square cubits per corner, which equals a grand total of sixteen million square cubits. Consequently, the open space is one quarter of the total extended boundary.48

Rav Ashi responded: What is the meaning of around? **Around the corners, i.e., an open space of this size is provided at each corner. As, if you do not so say, that the area of the corners is also called around, with regard to the burnt-offering, as it is written: “And they shall sprinkle the blood around upon the altar.”** (Leviticus 1:5), here, too, will you say that the blood must be sprinkled literally “around” the altar on all sides? The blood is sprinkled only upon the corners of the altar. Rather, what is the meaning of around? **Around the corners, i.e., the mitzva is to sprinkle the blood at the corners, and this is considered sprinkling blood “around upon the altar.”** Here too, with regard to the open space of the cities of the Levites, what is the meaning of around? **Around the corners.**

The Gemara returns to its previous statement that the open space around a city of the Levites is one quarter of the total extended boundary when the city is round. It questions this statement based upon the mishna’s ruling that the boundaries of a city are always delineated as a square. Rav Havivi from Mehoza said to Rav Ashi: But aren’t there the protrusions of the corners? How can there be a thousand cubits of open space on each side; when the city is squared, the corners of the square protrude into the open space, thus reducing its area?

Rav Ashi replied: We are dealing with a circular city. Rav Havivi responded: But haven’t they squared the city? Rav Ashi responded: Say that we say the following: **We view the city as if it were squared.** Do we actually add houses and square it? Although for the purpose of calculating the extended boundary we view the city as a square, in actuality the uninhabited sections are part of the open space.

Rav Hanilai from Mehoza said to Rav Ashi: Now, how much larger is the area of a square than the area of a circle? **One quarter.** Therefore, if we calculate how much area a circular city with a diameter of two thousand cubits gains when it is squared, does it add up to these eight hundred cubits mentioned above? The extra area added is only 667 minus one-third cubits.

Rav Ashi said to him: **This statement applies only to a circle enclosed within a square, as the area of a circle is three-quarters the area of the square around it. However, with regard to the additional diagonal** (alakhsona)49 space added in the corners of the square, more is required. As the Master said: Every cubit in the side of a square is one and two-fifths cubits in its diagonal. Based on this rule, the calculation is exact.
A city and its outside – לָרְ׳ֵּב לָעִיר and חוּצָר

Ravina’s position – שִׁיטַת רָבִי א

Moresha’s position – מֹרְשָׁא דְּ ַרְתָּא

Verse in the following manner: If there is only one city, the measure is added to the two cities only due to their proximity. In karpef HaLevi explains it completely differently: Ravina maintains that there is one quarter of the entire boundary. Consequently, the open space is one quarter of the area of corners. Thus, the open space is not provided. However, it is apparently equal to a known measure, that of seventy cubits.

Rav Ashi’s position – רָבִי אֵשִּׁי

The Ritva explains that the Rabbis provided. However, it is apparently equal to a known measure, that of seventy cubits. The author of the Shulchan Arukh generally follows the lenient opinion with regard to the laws of eiruv.

BACKGROUND

From the Greek λοξός, meaning a diagonal. The corresponding indigenous Aramaic word is karanzol, a city and a yard, which is the same as the area of the courtyard of the Tabernacle (Ra’avad).

LANGUAGE

The Shabbat limit of two thousand cubits around. And you shall measure from outside the city on the east side two thousand cubits” (Numbers 35:4–5). The Torah says: Provide a certain vacant space outside the city, and only afterward measure the two thousand cubits.

NOTES

We learned in the mishna: And the Rabbis say: They spoke of the addition of a karpef only with regard to the space between two adjacent cities. It was stated that the amora‘im disagreed with regard to this issue. Rav Huna said: One allocates a karpef to this city and a karpef to that city, so that the two cities together are granted a total of slightly more than 141 cubits. Hiyya bar Rav said: One allocates only one common karpef to the two of them.

The Gemara raises possible proofs for each opinion. We learned in the mishna: And the Rabbis say: They spoke of the addition of a karpef only with regard to the space between two adjacent cities. This appears to be a conclusive refutation of the opinion of Rav Huna, as it states that one karpef is allocated rather than two.

The Gemara answers that Rav Huna could have said to you in response to this difficulty: What is meant here by a karpef? It means the principle of a karpef. In actuality, one allocates a karpef to this city and a karpef to that city.

The Gemara comments: So, too, it is reasonable to explain the mishna in the following manner: From the fact that it teaches in the latter clause: If this city has seventy cubits and a remainder vacant on one side, and that city has seventy cubits and a remainder vacant on the adjacent side, and the two areas of seventy-plus cubits overlap, the karpef combines the two cities into one. This indicates that an area of seventy cubits and a remainder is added to each city. The Gemara concludes: Indeed, learn from this that this is the correct understanding of the mishna.

The Gemara asks: Let us say that this mishna is a conclusive refutation of the opinion of Hiyya bar Rav, that two adjacent cities are granted only one karpef. The Gemara answers that Hiyya bar Rav could have said to you:

MISHNA

One allocates a karpef to every city, i.e., the measure of a karpef, which is slightly more than seventy cubits, is added to every city, and the two thousand cubits of the Shabbat limit are measured from there; this is the statement of Rabbi Meir. And the Rabbis say: They spoke of the addition of a karpef only with regard to the space between two adjacent cities. How so? If this city has seventy cubits and a remainder vacant on one side, and that city has seventy cubits and a remainder vacant on the adjacent side, and the two areas of seventy-plus cubits overlap, the karpef combines the two cities into one.

And likewise, in the case of three villages that are arranged as a triangle, if there are only 141 1/3 cubits separating between the two outer villages, the middle village combines the three villages into one.

HALAKHA

A karpef to every city – לָרְ׳ֵּב לָעִיר יִשְׁמַע לָךְ. The Shabbat limit of two thousand cubits is calculated from the last residential house of the city, in accordance with the opinion of the Rabbis. However, some hold that an additional seventy cubits are added (Rema), in accordance with the lenient opinion of Rabbi Meir, as the halakha generally follows the lenient opinion with regard to the halakhot of eiruv (Shulhan Arukh, Orah Hayyim 398:8).

A karpef to this city and a karpef to that city – לָרְ׳ֵּב לָעִיר לָרְ׳ֵּב לָזֵר וְלָזֵר. If two cities are situated near each other, a karpef is allocated to each, for a total of 141 1/3 cubits. The halakha follows the opinion of Rav Huna over that of Hiyya bar Rav, since Rav Huna was older and was a greater Torah scholar. In addition, the halakha follows the lenient opinion with regard to the laws of eiruv (Shulhan Arukh, Orah Hayyim 398:7).
The Gemara continues to ask: If it is in accordance with the opinion of Rabbi Meir, didn’t we already learn in the first clause: One allocations a karpef to each city; this is the statement of Rabbi Meir? What need is there to mention Rabbi Meir’s opinion again?

The Gemara answers: It was necessary to mention his opinion again, as, if we had learned his opinion only from that first clause, I might have said that one allocates one karpef for one city and also one karpef for two cities. Therefore, the mishna teaches us that for two cities, one allocates two karpef areas.

And conversely, if the mishna had taught us this law only here, with regard to two cities, one might have said that only in that case is each city granted a separate karpef, because a smaller space between the two adjacent cities would be too crowded for the use of both cities. But there, with regard to one city, where the area of the city itself is not too crowded for the use of its residents, one might say that it is not given any karpef whatsoever. Therefore, it was necessary for the mishna to teach both clauses.

The Gemara tries again to adduce proof from the mishna, in which we learned: And likewise, in the case of three villages that are aligned in a row, if there is only 141½ cubits separating between the two outer ones, the middle village combines the three villages into one. At this point the Gemara understands that the mishna here is dealing with three villages arranged in a straight line. Therefore, it makes the following inference: The reason that the three villages are considered as one is only because there is a middle village, but were there no middle village, they would not be considered as one. This appears to be a conclusive refutation of the opinion of Rav Huna. According to Rav Huna, the two villages should be considered as one even without the middle village, due to the double karpef.

The Gemara rejects this argument: Rav Huna could have said to you: Wasn’t it stated with regard to that mishna that Rav Idi said that Rabbi Hanina said: It does not mean that the villages are actually aligned in a row of three villages but in a straight line. Rather, even if the middle village is off to one side and the outer villages are more than two karpef lengths apart, we see their spacing and make the following assessment: Any case where, if the middle village were placed between the other two so that they were three villages aligned in a row, there would be only a distance of 141½ cubits between one and the other, then the middle village turns the three villages into one. According to this explanation, the mishna can be understood even as a support for the opinion of Rav Huna.

With regard to this case, Rava said to Abaye: How much distance can there be between an outer village and the middle one, if the latter is still to combine the three villages into one? Abaye said to him: Two thousand cubits.

Rava replied: Wasn’t it you yourself who said: It is reasonable to rule in accordance with the opinion of Rava, son of Rabbi bar Rav Huna, who said: The Shabbat limit of a bow-shaped city is measured from the imaginary bowstring stretched between the two ends of the city, even if the distance between the center of the string and the center of the bow is more than two thousand cubits. Why shouldn’t the three villages in this case be considered a single village also, even if they are separated by more than two thousand cubits?
Abaye rejected the comparison: How can you compare? There, in the case of the bow-shaped city, there are houses that combine the city into a single unit, whereas here, there are no houses linking the outer villages. Therefore, if two villages are separated by more than two thousand cubits, the measure of the Shabbat limit, they cannot be considered a single entity.

And Rava said to Abaye: How much distance can there be between one outer village and the other outer village? Abaye expressed surprise at this question: How much distance can there be between them? What is the practical difference to you? Any case where, if the middle village were placed between them, there would be only a distance of 141/3 cubits between one and the other, the middle village turns the three villages into one. Therefore, the critical detail is not the distance between the outer villages but the size of the middle village.

Rava continued his line of questioning: Is this true even if the distance between the two outer villages is four thousand cubits? Abaye said to him: Yes. Rava asked: Didn’t Rav Huna say the following with regard to a city shaped like a bow: If the distance between its two ends is less than four thousand cubits, one measures the Shabbat limit from the imaginary bowstring stretched between the two ends of the bow; and if not, one measures the Shabbat limit from the bow itself? This indicates that even if there is an uninterrupted string of houses linking the two ends of the city, if the two ends are separated by more than four thousand cubits, the distance is too great for it to be considered a single city.

Abaye said to him: There, in the case of the bow-shaped city, there is no room to say: Fill it in,4 as there is nothing with which to fill in the empty space between the two ends of the city. However, here, in the case of the villages, there is room to say: Fill it in, as the middle village is seen as though it were projected between the two outer villages, and therefore all three combine into a single village.

Rav Safra said to Rava: With regard to the people of the city of Akistefon, for whom we measure the Shabbat limit from the far end of the city of Ardeshir, and the people of Ardeshir, for whom we measure the Shabbat limit from the far end of Akistefon,5 as though the two settlements were a single city; isn’t there the Tigris River, which separates them by more than 141/3 cubits? How can two cities that are separated by more than two karpaf-lengths be considered a single entity?

Rava went out and showed Rav Safra the foundations of a wall of one of the cities, which were submerged in the Tigris River at a distance of seventy cubits and a remainder from the other city. In other words, the two cities were in fact linked through the remnants of a wall submerged in the river.

MISHNA One may measure a Shabbat limit only with a rope fifty cubits long, no less and no more, as will be explained in the Gemara. And one may measure the limit only at the level of one’s heart, i.e., whoever comes to measure the limit must hold the rope next to his chest.6

If one was measuring the limit and he reached a canyon or a fence, the height of the fence and the depth of the canyon are not counted toward the two thousand cubits; rather, he spans it and then resumes his measurement.7 Two people hold the two ends of the rope straight across the canyon or the fence, and the distance is measured as though the area were completely flat. If one reached a hill, he does not measure its height; rather, he spans the hill as if it were not there and then resumes his measurement.

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